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Tech Notes

Linear Regression Bias – Phase Lag & Noise

An often-overlooked mathematical complication can affect slope estimates

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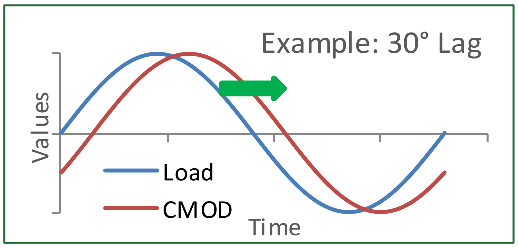
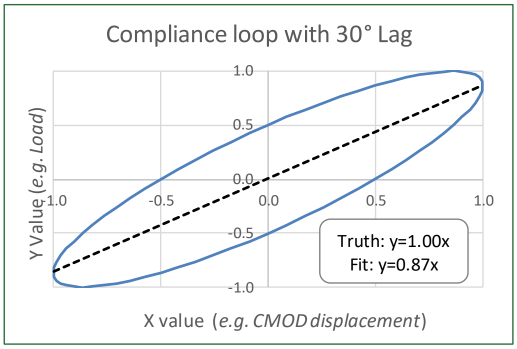
Regression Bias

Many applications in mechanical testing use slope estimates of XY data, *e.g.* stress-vs-strain for calculation of elastic modulus in tensile testing, and load-vs-CMOD for compliance calculation in fatigue testing. If a common best fit line (*least squares regression*) method of slope calculation is used, phase lag and/or noise in the raw data can result in a bias (*error*) in the calculated result due to the regression method itself. Some regression methods are less susceptible to regression bias, *but the regression method used by the test software may not be evident to the user.*

Phase Lag – impacts on cyclic testing



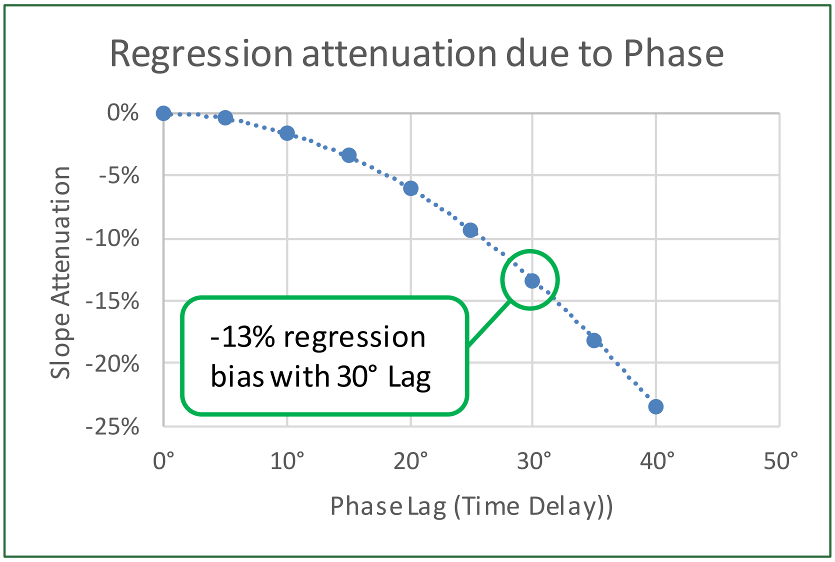
Phase lag (*time delay*) can result from the intentional or inadvertent use of mismatched signal filters on the two data channels used for the calculation and can bias the slope measurement. In this example, a 30° lag on the CMOD channel relative to load has resulted in a 13% error in the calculated compliance; *the crack length will be overestimated*.



This *apparent* attenuation is independent from and adds to any *actual* attenuation caused by a signal filter.

> *If filter attenuation or phase lag regression bias are impacting test results, it is advisable to use similar filters on both channels.*

Due to the potential sources of error in the calculation of best-fit lines for cyclic data as described here, the researcher is advised to select a test profile and filters which yield sufficiently small errors. A maximum phase lag of 8° may be recommended, to keep potential regression bias errors below 1%.



https://en.wikipedia.org/wiki/Linear_regression

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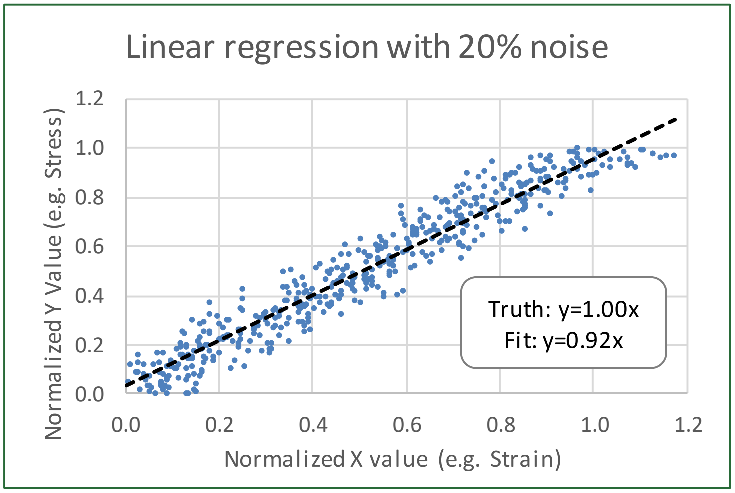
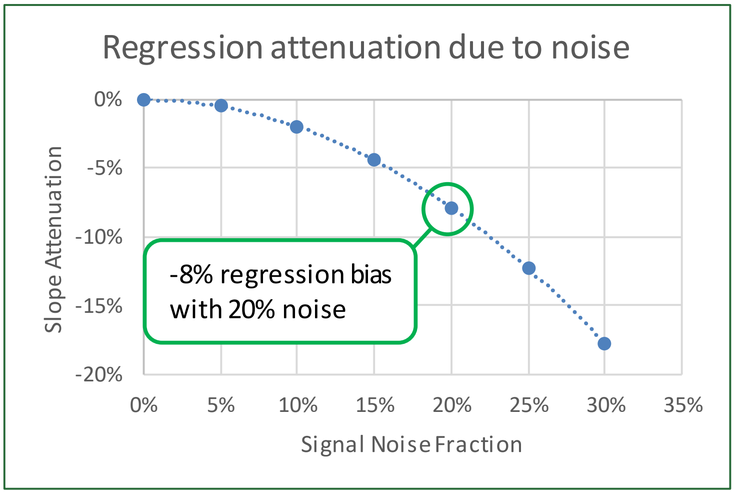
Signal Noise – impacts on small strain measurements

Another source of bias error due to least-squares regression is *signal noise*. Random scatter in the raw X or Y data can affect measurements where the measured values are small relative to the noise in the signals. This can be most relevant when measurement ranges are small.

In the example below, the *signal-to-noise ratio* is 5:1, a very noisy signal. The slope of the line (*i.e., elastic modulus*) has been underestimated by 8% by the line fit.





> *If signal noise regression bias is impacting test results, it is advisable to improve the signal-to-noise ratio.*

Signal noise bias can be mitigated by using a bias-free estimator, or by improving the signal-to-noise ratio. Increasing the measurement range (*e.g., calculate modulus over a larger strain range*), or decreasing the noise, can be effective in improving the signal-to-noise ratio. Signal noise can often be reduced, depending on the root cause, by increasing the gauge length, applying a suitable filter, mitigating vibration, and other mechanisms.

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